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CP PUBLICATION

SHORT TRICKS

Mathematics

for JEE (Main & Adv)

Sunil Gupta
Career Point Editorial

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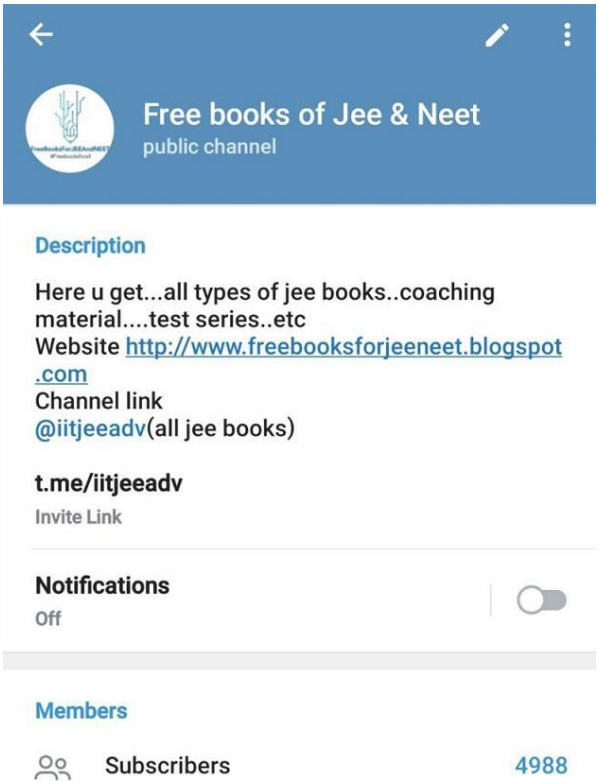
SHORT TRICKS
Mathematics
for JEE (Main & Advanced)



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Differentiation

KEY CONCEPTS

1. Differentiation

The rate of change of quantity y with respect to another quantity x is called the derivative or differential coefficient of y with respect to x.

The process of finding derivative of a function is called differentiation.

2. Differential Coefficient

Let $y = f(x)$ be a continuous function of a variable quantity x. Let δx be an arbitrary small change in the value of x and δy be the corresponding change in y then $\lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ if it exists, is called the differential coefficient of y with respect to x and it is denoted by $\frac{dy}{dx}$. y', y_1 or Dy.

So, $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$

$\therefore \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{\delta x}$

This method is known as first principle or ab initio or delta method.

3. Differential Coefficient of Some Standard Function

(i) $\frac{d}{dx}(\text{constant}) = 0$

(ii) $\frac{d}{dx}(ax) = a$

(iii) $\frac{d}{dx}(x^n) = nx^{n-1}$

(iv) $\frac{d}{dx}e^x = e^x$

(v) $\frac{d}{dx}(a^x) = a^x \log_e a \quad (a > 0, a \neq 1)$

(vi) $\frac{d}{dx}(\log_e x) = \frac{1}{x} \quad (x > 0)$

(vii) $\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a} = \frac{1}{x}(\log_a e)$

(viii) $\frac{d}{dx}(\sin x) = \cos x$

(ix) $\frac{d}{dx}(\cos x) = -\sin x$

(x) $\frac{d}{dx}(\tan x) = \sec^2 x$

(xi) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(xii) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(xiii) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(xiv) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, -1 < x < 1$

(xv) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}, -1 < x < 1$

(xvi) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(xvii) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

(xviii) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}, |x| > 1$

(xix) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

(xx) $\frac{d}{dx}(\sinh x) = \cosh x$

(xxi) $\frac{d}{dx}(\cosh x) = \sinh x$

(xxii) $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

(xxiii) $\frac{d}{dx}(\coth x) = -\operatorname{cosec} h^2 x$

(xxiv) $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

(xxv) $\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$

(xxvi) $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

(xxvii) $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}, x > 1$

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(xxviii) $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$

(xxix) $\frac{d}{dx}(\coth^{-1}x) = \frac{1}{x^2-1}$, $|x| > 1$

(xxx) $\frac{d}{dx}(\text{sech}^{-1}x) = -\frac{1}{x\sqrt{1-x^2}}$, $(0 < x < 1)$

(xxxi) $\frac{d}{dx}(\text{cosech}^{-1}x) = -\frac{1}{|x|\sqrt{x^2+1}}$, $x \neq 0$

(xxxii) $\frac{d}{dx}e^{ax}\sin bx) = e^{ax}(a\sin bx + b\cos bx) = \sqrt{a^2+b^2}\, e^{ax}\sin(bx + \tan^{-1} b/a)$

(xxxiii) $\frac{d}{dx}(e^{ax}\cos bx) = e^{ax}(a\cos bx - b\sin bx) = \sqrt{a^2+b^2}\, e^{ax}\cos(bx + \tan^{-1} b/a)$

4. Some Theorems on Differentiation

Theorem I $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$, where k is a constant

Theorem II $\frac{d}{dx}[f_1(x) \pm f_2(x) \pm f_3(x) \pm ....]$

$= \frac{d}{dx}[f_1(x)] \pm \frac{d}{dx}[f_2(x)] \pm \frac{d}{dx}[f_3(x)] + ...$

Theorem III $\frac{d}{dx}[f(x).g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$

Theorem IV $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$

Theorem V **Derivative of the function of the function (CHAIN RULE).**

If 'y' is a function of 't' and 't' is function of 'x' then

$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$

Theorem VI **Derivative of parametric equations.**

If $x = \phi(t)$, $y = \psi(t)$ then

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\psi'(t)}{\phi'(t)}$

Theorem VII **Derivative of a function with respect to another function.**

If f(x) and g(x) are two functions of a variables x, then differentiation of f(x) w.r.t. g(x) can be written as

$\frac{d[f(x)]}{d[g(x)]} = \frac{d}{dx}[f(x)] / \frac{d}{dx}[g(x)]$

5. Differentiation of Implicit Functions

If in an equation, x and y both occurs together i.e. f(x, y) = 0 and this equation can not be solved either for y or x, then y (or x) is called the implicit function of x (or y).

For example $x^3 + y^3 + 3axy + c = 0$, $x^y + y^x = a^b$ etc.

Working Rule for Finding the Derivative

First Method :

(ia) Differentiate every term of f(x,y) = 0 with respect to x.

(bii) Collect the coefficients of $\frac{dy}{dx}$ and obtain the value of $\frac{dy}{dx}$.

Second Method :

If f (x, y) = constant, then $\frac{dy}{dx} = -\frac{\partial f / \partial x}{\partial f / \partial y}$

where $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are partial differential coefficients of f(x, y) with respect to x and y respectively.

6. Differentiation of $[f(x)]^{g(x)}$ type Functions

$y = [f(x)]^{g(x)}$

$\log y = g(x) \log [f(x)]$

$\frac{1}{y}.\frac{dy}{dx} = \frac{d}{dx} g(x).\log [f(x)]$

$\frac{dy}{dx} = [f(x)]^{g(x)}. \left\{\frac{d}{dx}[g(x) \log f(x)]\right\}$

7. Differentiation by Trigonometrical Substitutions

Some times before differentiation, we reduce the given function in a simple form using suitable trigonometrical substitutation.

Function	Substitution`
(i) $\sqrt{a^2 - x^2}$	$x = a \sin\theta$ or $a \cos\theta$
(ii) $\sqrt{x^2 + a^2}$	$x = a \tan\theta$ or $a \cot\theta$
(iii) $\sqrt{x^2 - a^2}$	$x = a \sec\theta$ or $a \operatorname{cosec}\theta$
(iv) $\sqrt{\frac{a - x}{a + x}}$	$x = a \cos 2\theta$
(v) $\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$	$x^2 = a^2 \cos 2\theta$
(vi) $\sqrt{ax - x^2}$	$x = a \sin^2\theta$
(vii) $\sqrt{\frac{x}{a + x}}$	$x = a \tan^2\theta$
(viii) $\sqrt{\frac{x}{a - x}}$	$x = a \sin^2\theta$
(ix) $\sqrt{(x - a)(x - b)}$	$x = a \sec^2\theta - b \tan^2\theta$
(x) $\sqrt{(x - a)(b - x)}$	$x = a \cos^2\theta + b \sin^2\theta$

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8. Differentiation of Infinite Series

(i) If $y = \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x) + \ldots \infty}}}$ then

$\Rightarrow y = \sqrt{f(x) + y}$

$\Rightarrow y^2 = f(x) + y$

$\Rightarrow 2y \frac{dy}{dx} = f'(x) + \frac{dy}{dx} \quad \therefore \frac{dy}{dx} = \frac{f'(x)}{2y-1}$

(ii) If $y = f(x)^{f(x)^{f(x)\ldots\infty}}$ then $y = f(x)^y$.

$\therefore \log y = y \log [f(x)]$

$\frac{1}{y}\frac{dy}{dx} = \frac{y.f'(x)}{f(x)} + \log f(x) . \left(\frac{dy}{dx}\right)$

$\therefore \frac{dy}{dx} = \frac{y^2 f'(x)}{f(x)[1 - y \log f(x)]}$

(iii) If $y = f(x) + \frac{1}{f(x) + \frac{1}{f(x) + \frac{1}{f(x)\ldots\infty}}}$ then $\frac{dy}{dx} = \frac{yf'(x)}{2y - f(x)}$.

9. Differentiation of Determinants

To differentiate a determinant, differentiate one row (or column) at a time, keeping others unchanged.

If $\Delta(x) = \begin{vmatrix} A(x) & B(x) \\ C(x) & D(x) \end{vmatrix}$

then $\frac{d}{dx}\Delta(x) = \begin{vmatrix} A'(x) & B'(x) \\ C(x) & D(x) \end{vmatrix} + \begin{vmatrix} A(x) & B(x) \\ C'(x) & D'(x) \end{vmatrix}$

10. n^{th} Derivatives of Some Standard Functions

(i) $\frac{d^n}{dx^n} \sin(ax + b) = a^n \sin\left(\frac{n\pi}{2} + ax + b\right)$

(ii) $\frac{d^n}{dx^n} \cos(ax + b) = a^n \cos\left(\frac{n\pi}{2} + ax + b\right)$

(iii) $\frac{d^n}{dx^n} (ax + b)^m = \frac{m!}{(m-n)!} a^n (ax + b)^{m-n}$, where $m > n$

(iv) $\frac{d^n}{dx^n} (\log (ax + b)) = \frac{(-1)^{n-1}(n-1)! a^n}{(ax + b)^n}$

(v) $\frac{d^n}{dx^n} (e^{ax}) = a^n e^{ax}$

(vi) $\frac{d^n(a^x)}{dx^n} = a^x (\log a)^n$

(vii) $\frac{d^n}{dx^n} (e^{ax} \sin (bx + c)) = r^n e^{ax} \sin (bx + c + n\phi)$ where $r = \sqrt{a^2 + b^2}$; $\phi = \tan^{-1}\frac{b}{a}$

(viii) $\frac{d^n}{dx^n} (e^{ax} \cos (bx + c)) = r^n e^{ax} \cos (bx + c + n\phi)$

where $r = \sqrt{a^2 + b^2}$; $\phi = \tan^{-1}\frac{b}{a}$

SHORT TRICKS

Trick -1 If $\sqrt{1-x^{2n}} + \sqrt{1-y^{2n}} = a^n(x^n - y^n)$, then $\frac{dy}{dx} = \frac{\sqrt{1-y^{2n}}}{\sqrt{1-x^{2n}}} \frac{x^{n-1}}{y^{n-1}}$

Q.1 If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then $\frac{dy}{dx}$ is equal to -

(A) $\sqrt{1-x^2}\sqrt{1-y^2}$ (B) $\sqrt{\frac{1-y^2}{1-x^2}}$ (C) $\sqrt{\frac{1-x^2}{1-y^2}}$ (D) $\sqrt{\frac{1+y^2}{1+x^2}}$

Sol. **[B]**

Proper Method

Put $x = \sin\theta$, $y = \sin\phi$

$\sqrt{1-\sin^2\theta} + \sqrt{1-\sin^2\phi} = a(\sin\theta - \sin\phi)$

$\Rightarrow \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$\Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = a\left(2\cos\frac{\theta+\phi}{2}.\sin\frac{\theta-\phi}{2}\right)$

$\Rightarrow \cos\left(\frac{\theta+\phi}{2}\right) = a\sin\left(\frac{\theta-\phi}{2}\right)$ $(\because \theta + \phi \neq \pi)$

$\Rightarrow \cot\left(\frac{\theta-\phi}{2}\right) = a$

$\Rightarrow \theta - \phi = 2\cot^{-1} a$

$\Rightarrow \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$

Differentiation w.r.t x

$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Short Trick

Put n = 1 in standard Result

$\frac{dy}{dx} = \sqrt{\frac{1-y^{2n}}{1-x^{2n}}}\ \frac{x^{n-1}}{y^{n-1}}$

$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

Q.2 If $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3(x^3 - y^3)$, then $\frac{dy}{dx}$ is equal to -

(A) $\sqrt{\frac{1-y^6}{1-x^6}}.\frac{x^3}{y^3}$ (B) $\sqrt{\frac{1-y^6}{1-x^6}}.\frac{x^2}{y^2}$ (C) $\sqrt{\frac{1+y^6}{1+x^6}}.\frac{x^3}{y^3}$ (D) $\sqrt{\frac{1+y^6}{1+x^6}}.\frac{y^2}{x^2}$

Sol. **[B]**

Proper Method

Put $x^3 = \sin\theta$, $y^3 = \sin\phi$

$\sqrt{1-\sin^2\theta} + \sqrt{1-\cos^2\theta} = a^3(\sin\theta - \sin\phi)$

$\Rightarrow \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$\Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = a\left(2\cos\frac{\theta+\phi}{2}.\sin\frac{\theta-\phi}{2}\right)$

$\Rightarrow \cos\left(\frac{\theta-\phi}{2}\right) = a\sin\left(\frac{\theta-\phi}{2}\right)$

Short Trick

Put n = 3 in standard Result

$\frac{dy}{dx} = \sqrt{\frac{1-y^6}{1-x^6}} \cdot \frac{x^2}{y^2}$

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$\Rightarrow \cot\left(\frac{\theta-\phi}{2}\right) = a$

$\Rightarrow \theta - \phi = 2\cot^{-1}a$

$\Rightarrow \sin^{-1}x^3 - \sin^{-1}y^3 = 2\cot^{-1}a$

diff. w.r.t x

$\Rightarrow \frac{1}{\sqrt{1-x^6}}.3x^2 - \frac{1}{\sqrt{1-y^6}}.3y^2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^6}{1-x^6}}.\frac{x^2}{y^2}$

Trick -2 Method of substitution

Q.3 Let f(x) be a polynomial function of second degree. If f(1) = f(–1) and a, b, c are in A.P. then f'(a), f'(b) and f'(c) are in- **[AIEEE 2003]**

(A) Arithmetic - Geometric Progression (B) A.P.

(C) G.P. (D) H.P.

Sol. **[B]**

Proper Method

$\because$ f(x) is polynomial of second degree

$\therefore$ let $f(x) = Ax^2 + Bx + C$

$\therefore$ f(1) = f(–1)

$\Rightarrow A + B + C = A - B + C$

$\Rightarrow 2B = 0 \Rightarrow B = 0$

$\therefore f(x) = Ax^2 + C$

$f'(x) = 2Ax + C$

$\therefore f'(a) = 2Aa + C$

$f'(b) = 2Ab + C$

$f'(c) = 2Ac + C$

$\because$ a, b, c are in A.P.

$\therefore$ 2Aa, 2Ab, 2Ac, are also in A.P.

& 2Aa + C , 2Ab + C, 2Ac + C .....

Short Trick

Let $f(x) = x^2$

$f'(x) = 2x$

$\therefore f'(a) = 2a, f'(b) = 2b, f'(c) = 2c$ are in A.P.

Q.4 If $y = \sin^2\alpha + \cos^2(\alpha+\beta) + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$, then $\frac{d^3y}{d\alpha^3}$ is-

(A) $\frac{\sin^3(\alpha+\beta)}{\cos\alpha}$ (B) $\cos(\alpha+3\beta)$ (C) 0 (D) none of these

Sol. **[C]**

Proper Method

$y = \sin^2\alpha + \cos^2(\alpha+\beta) + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$

$= \sin^2\alpha + \cos(\alpha+\beta)[\cos(\alpha+\beta) + 2\sin\alpha\sin\beta]$

$= \sin^2\alpha + \cos(\alpha+\beta)[\cos\alpha\cos\beta - \sin\alpha\sin\beta + 2\sin\alpha\sin\beta]$

$= \sin^2\alpha + \cos(\alpha+\beta)[\cos(\alpha-\beta)]$

$= \sin^2\alpha + \cos(\alpha+\beta)\cos(\alpha-\beta)$

$= \sin^2\alpha + \cos^2\beta - \sin^2\alpha$

$y = \cos^2\beta$

$\frac{dy}{d\alpha} = 0 \quad \therefore \frac{d^3y}{d\alpha^3} = 0$

Short Trick

Put $\beta = 0 \Rightarrow y = (\sin^2\alpha + \cos^2\alpha) + 0 = 1$

$\frac{dy}{d\alpha} = 0$

$\frac{d^3y}{d\alpha^3} = 0$